Final Report

Probabilistic methods for communication networks with mobile relays

Leibniz-Institute: Weierstrass Institute for Appplied Analysis and Stochastics, Berlin

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1. General Information

Responsible: Prof. Dr. Wolfgang König (Weierstrass Institute Berlin, WIAS), head of Research Group 5 *Interacting Random Systems*

Further PIs: Dr. Robert Patterson (WIAS, RG 5), Prof. Dr.-Ing. Rolf Kraemer and Dr. Marcin Brzozowski (Innovations for High Performance Microelectronics (IHP) Frankfurt / Oder)

Funded postdocs at WIAS: Christian Hirsch (Sep. 2014 – Sept. 2016), Benedikt Jahnel (Jan. 2015 – June 2017), Paul -Keeler (Sept. 2014 – June 2017).

Homepage: `http://www.wias-berlin.de/research/lgs/lg4/index.jsp?lang=0`

Repository: `http://doi.org/10.20347/WIAS.DATA.2`

2. Executive Summary

In joint efforts of probabilists and system engineers, spatial telecommunication models of next generation functionalities are deviced and rigorously analysed. The main source of randomness is the location of the user devices, another one comes from the time instances of message transmission. Important features are considered like capacity constraints (a relay can cope only with a bounded number of messages at at time), interference of many message transmissions (measrued in terms of the signal-to-interference ratio), navigation schemes (the messages are sent into a certain direction), mobility (the user devices move randomly in space). In order to be able to apply rigorous mathematical methods, two types of limiting settings are imposed, the high-density limit (many users in a bounded area) and the thermodynamic limit (many users in a large area, such that their density is constant). The main quantities that were considered are the percentage of satisfied users (having a connection, or being able to transmit their message successfully) and the spatial description of the main message flow. The main focus is put on the analysis of the probabilities of frustration events, which are very rare events of a bad service quality, e.g., that too small a percentage of users is satisfied. Here the mathematical theory of large deviations proved useful, which provides a framework to describe the smallness of the probability and of the most likely situation that leads to it. Finally, a simulation tool was developed that makes the rays of message transmission visible, including reflections, at the example of the Hausvogteiplatz in Berlin-Mitte.

3. Starting Points and Goals

This is a resarch about the performance of a spatial telecommunication system of the next generation (5G), using modeling and rigorous mathematical analysis. The goal is to design and investigate a novel hybrid type of a spatial network that consists of fixed base stations and a mobile ad-hoc systems around each of the base stations. The idea is that the number of expensive base stations can be made smaller in this way, compared to a system where the signals go directly from the users to the base station; the multihop functionality extends the reach of the base station. For an extended analysis, we proposed to augment the system by auxiliary mobile relays, installed by the operator, but not by users.

One of the fundamental aspects of the model is the use of probability, as the locations of the users and the relays is supposed to be random (and independent). It was planned to address the most basic questions about the service quality of the system, like *connectivity* (what is the probability that the locations of the users are such that messages can travel sufficiently far?), *interference* (what is the probability that there are locally too many messages transmitted, such that too many of them cannot be filtered out of the superposition of all the message signals?) and *capacity* (what is the probability that too many messages choose the same relay for a hop, such that it is overweighted and cannot work properly?).

In the modeling part of the project, the model is designed and set up in accordance with technical requirements and considerations of modern functionalities. Here the development of a simulation tool was planned that takes into account the above important questions. However, the main work was planned on mathematical modeling using tools from the probabilistic theory of *stochastic geometry* and on extending them with rigorous mathematical work. Particular attention is payed to the question of *frustration probabilities*, i.e., the probabilities of events of a bad service quality, and to the description of the most probable origin and reason for this.

Another direction that was planned was the *randonm mobility* of the users in the system and its impact on the above three fundamental quantities. Further questions concern the effect of additional, immobile relays, which should enhance connectivity.

An important aspect of the project is a development of a mode of cooperation of the two research approaches (rigorous probability and system engineering and simulation, respectively) that would lead to results of the two approaches that can be compared to each other and would coincide in some way. This is a difficult task, as the working mode of the probabilistic part of the project is based on limit theorems that do not depend on units and give only rough rules of thumbs, while the simulation work depends on the right choices of the units. Both approaches need to work on strongly simplified models, hence the difficulty would be to develop models that are sufficiently simple for analyses of both kinds and still capture the main essence of the effect to be studied.

4. Activities

Joint discussions. One of the most fundamental goals of the entire project is to develop a mutual understanding of system engineers and probabilists and to learn the respective languages and to jump over the language barrier. This is not an easy task, since the ansatzes, the methods and the goals are rather different between these two research cultures. Nevertheless, we think that, over the running period of three years, the two subgroups of the project managed to overcome this difficulty and to turn it into a fruitful collaboration. The WIAS group and the IHP group jointly discussed (in about four meetings per year) the details of the pecularities and properties of a hybrid system of the above characterised type and decomposed it into mesoscopic and macroscopic parts. The goal of these discussions was to reach a mathematical formulation and modeling of such systems that would be amenable to a rigorous probabilistic analysis on the one hand and to a simulation analysis on the other. Many of the details of the discussions concentrated on the right choices of the functionalities, like the maximal number of hops that one should allow before the message reaches the base station and questions like whether or not it should be possible that messages reach the target without arriving at a base station. Furthermore, the pros and cons of a number of random mobility models were discussed. These discussions shaped and strongy influenced the mathematical modeling of the rigorous probabilistic part of the project and the precise questions that were attacked.

Mathematical and simulation research. The right coice of the mathematical model was shaped in the above described discussions, and the WIAS group worked then on the part of the precise formulations of the model, the questions and the theorems and, of course, derived and worked out the proofs with mathematical rigour. The IHP group developed simulation tools for an adequate model for a descrption of the rays of the trajectories of the messages with relaying, in the example of the spatial area of the Hausvogteiplatz.

Workshop. The WIAS and the IHP jointly organised in November 2016 at WIAS a workshop on *Probabilistic Methods in Telecommunication* as part of their joint activities in this project, see `http://www.wias-berlin.de/workshops/PMT16/`. The format consisted of two minicourses (three times 90 minutes) in the mornings, given by world experts on the probabilistic modelling of spatial telecommunication systems, and eight research talks in the afternoons. The invited speakers stemmed both from the fields of information science and of applied probability. A poster session rounded the success of the workshop, which was attended by approximately 30 researchers.

Inclusion of young scientists. The professoral members of the team, Prof. König (at the Mathematical Institute of the TU Berlin) and Prof. Rolf Kraemer (at the Mathematical Institute of the Brandenburgische Technische Universität Cottbus) involved a number of young people into the subject by initiating and supervising bachelor and master theses and PhD projects. The most advanced of these youg people occaionally took part in the discussions.

5. RESULTS

Frustration probabilities. One of the main directions of research of the project was devoted to the evaluation of the probabilities of very improbable events of a bad service quality. This was done by exploiting the benefits of a probabilistic theory called the *theory of large deviations*, which has hardly before been used in the framework of telecommunication.

One of the main examples is the following; see [HJPP17a]. Imagine that a large number of users are randomly located in a cell with one base station. Each of the users has a message that should be delivered to the base station, either directly or with one hop via another user inbetween. One instance that influences the doability is the question whether or not the distance between the transmitter and the base station or the used relay is small enough, and another one is the interference, i.e., whether or not the used relay is too overloaded by this function. In order to analyse the situation in terms of a mathematical theorem, one considers the limiting scenario of a high-density system, i.e., the number of users per unit area tends to infinity. The main considered object is the limiting cloud of users, and according to a law of large numbers, the average percentage of satisfied users is some quantity, hopefully close to one (but difficult to identify, though).

We now consider the event that the actual percentage of satisfied users is below a threshold that is strictly away from the limit. In order to handle this event, we look at the event that the cloud converges towards some given spatial density that satisfies that satisfactory constraint (here some serious continuity problems arise). The exponential decay rate of the probability of this event is quantified in terms of what is called a large-deviations principe (LDP). The rate itself is expressed in terms of an analytic formula (indeed, an entropy). Now the rate of the probability of the event of bad service quality can be expressed as the infimum over all limiting densities of the rate function, and the minimising density should be the most likely spatial distribution of the users that lead to this event.

This mathematical approach using large deviations has the two good points that it gives precise information about the smallness of the probability and even about the most likely user distribution that is responsible for the bad service quality. In the situation that one is interested in (rare events), the use of simulations is computationally so tremendously cumbersome that this strategty can be considered undoable. Instead, the formulas that the tehory of large deviaitons provides give a strong input into an improved application of *importance sampling*, see [HJPP16]. However, a mathematical analysis of this information is rather difficult and would need some simulation tools, this has been carried out only under additional simplifying assumptions in our project.

Limiting settings. Above, we illustrated the use of the theory of large deviations in a limiting setting of the telecommunication setting that is characterised by a high spatial density of users. Another limiting setting in which the above approach is useful and was carried out in the project is the setting where the considered communication area and the number of users grows unboundedly, but the density (number of users per unit area) is kept fixed. This is a limit that is called in Physics the *thermodynamic limit*. There is a mathematical theory about that, but the arising formulas are even less explicit, and there is not much hope for finding simple formulas unless in simplified formulas with the help of simulations.

Capacity constraints. In [HJP18, HJ17], the following important aspect was studied: constraints that come from an overloading of nodes because too many message trajectories chose them as a relay. As the claim is that the situation is described explicitly and with mathematical rigour, the model has to be extremely simple: each relay can cope only with one message at at time. The users are randomly located like a Poisson point process, while the relay nodes are given at deterministic places. In this model, which has a finite time axis $[0, T]$, each message from any node chooses randomly a node as a relay after a random waiting time. However, if the chosen relay already received another message, then it is occupied and the attempt by the second message fails. In [HJP18], the possibility that the relay is later again vacant is not considered, as this is mathematically much more demanding (a certain monotonicity is not present, which renders the mathematical analysis much simpler). This additional feature is inserted in the model considered in [HJ17]. Now we consider a user that sends out a message *frustrated* if his message is not successfully transmitted to the relay becaus the relay is occupied. The main result of these papers is the derivation of a large-deviation analysis of the description of the frustrated users in the time-space setting in the high-density limit (see above).

Navigation scheme and traffic flow. In [HJPP17b] the additional feature of a navigation scheme was introduced, a particular choice of the direction of the hop of the message, and the main flow of the messages was described as a function of that scheme and the other main parameters, the spatial density of the nodes (locations of user devices) and the rate at which messages are transmitted. The thermodynamic limit (see above) was considered, and a formula for the amount of traffic flow in a given direction was derived. In a natural particular case, the formula was illustrated.

Random mobility. One of the most important, but also most difficult to model and to rigorously analyse, features is the random movement of the devics (users). In some, but not all, results of the project, the model and the proof method allows for a random mobility of all the users. More precisely, instead of static locations of the users, they are represented as random trajectories, and all statistical properties that came from the trajectories are contained in the model. The trajectory had an influence of an additional parameter in the description; its influence was more or less restricted to singel time instances, over which all other effects were integrated. Due to the high mathematical complexity, it was not possible during the project running time to set up and analyse a model in which the movement of the devices interacts on the same time scale as the message transmission and therefore gives rise to additional effects like delay, congestion, or a more effective space-time picture. We believe that a proper mathematical handling of this is a comprehensive task and would need much more efforts and time.

Simulation. The group also provided a simulation of the rays of a signal emitted at various places of the Huasvogteiplatz (the location of the WIAS), under reflection at the surfaces of the buildings. The input data stem form the (public) Berlin3D-Modell, see `http://www.businesslocationcenter.de/en/downloadportal`. The group chose the targeted excerpt and slightly adapted them; the result is publicly available at the project repository `http://doi.org/10.20347/WIAS.DATA.2`. These data, together with the results of the simulations, are also available there. The grop used the tool RaLaNS, which is maintained and secured by the University of Osnabrück and made publicly available on the page `http://sys.cs.uos.de/RaLaNS/`. See the appendix, Section III.B, of [K+18] for an explanation of how the simulation was done and what its merits are.

Continuation. There is a continuing impact of the project on ongoing research. The PI and one of the project postdocs, Benedikt Jahnel, were successful in obtaining a funding of a PhD position in the ECMath project MI11: “Data mobility in ad-hoc networks: Vulnerability and security” in the framework of MATHEON supported by Einstein Foundation Berlin (running time June 2017 – December 2018). An extension of this project has been recently granted in the Cluster of Excellence “MATH+” for three more years. These two projects together fund a PhD project at WIAS in this field. One of the project postdocs, Christian Hirsch (now at Aalborg University, Denmark) continues to be an external collaborator of the WIAS group. This research is carried out in close collaboration with Orange S.A., see Section6.

6. Industrial Impact

The results obtained in this project were disseminated also by talks on conferences and workshops. At one of these wokshops, a talk given by Christian Hirsch was the starting point for a contact with Orange S.A. (former France Télécom, Paris). This contact between Orange and WIAS turned out to be rather fruitful for both sides and lead to a long-term collaboration. There have been three one-year contracts between Orange and WIAS on the subject of the project yet, and more are likely to follow. Among others, the papers [C+18, CHJ18] stem from that collaboration; but we do not include this line of research in the present report.

7. Contribution from Externals

The core of the work of the project was done in joint work by exclusively the members of the group; externals were not involved other than by discussions on meetings and workshops. Since one postdoc (Christian Hirsch) left the group for Munich in Fall 2016, he may be considered external from that time on, but he kept on going working with the group and continues to do so.

8. Theses

A number of young researchers on various levels of education have been brought into contact with the subject of this project and participated in the research. Here are the most important activities around the project in view of the integration of young scientists:

- PhD project András Tóbiás, funded by the Berlin Mathematical School July 2016 – June 2019, subject: random message routing in highly dense telecommunication systems, connectivity in random street systems, see [KT17a, KT17b].
- PhD project Alexander Hinsen (né Wapenhans), funded by Matheon Project MI11 in the Einstein Center Berlin, June 2017 – December 2018 and Project EF4-1 in the Cluster of Excellence "MATH+" 2019 – 21, supervised by Benedikt Jahnel and Wolfgang König; subject: propagation of malware in wireless telecommunication systems.
- Students' seminar at TU Berlin in Summer Term 2015 on "Random Communication networks", supervised by König and Hirsch.
- Lecture *Probabilistic Methods in Telecommunication* given by Benedikt Jahnel and Wolfgang König at TU Berlin in the Summer Term 2018; authors of corresponding lecture notes.

- Students' seminar at TU Berlin in Summer Term 2018 supervised by König, Jahnel and Tobiás accompanying the above lecture on the same subject.

Here is the list of titles of Bachelor (B) and Master (M) theses that were supervised by PI König at TU Berlin (partially unter assistance of the project postdocs Jahnel and Hirsch) during the running time (with finishing date); they are strongly connected with the research done in the project.

- Konnektivität im Booole'schen Modell zu einem Gitter bei beschränkter Reichweite (B, October 2014)
- Optimales Routing auf einem Poisson-Delaunay-Graphen (B, September 2015)
- Perkolation in SINR-Graphen (B, September 2015)
- Highly dense mobile communication networks with random fandings (M, May 2016)
- Gibbs-Maß für Trajektorien von Nachrichten in einem Kommunikationsnetzwerk (M, June 2016)
- Interference in ad-hoc telecommunication systems in the high-density limit (M, August 2016)
- Signal-to-Interference-Ratio in Kommunikationsnetzwerken mit hoher Dichte (B, December 2016)
- Perkolation mit Interferenz bei beschränkter Sprungzahl (B, March 2017)
- Modellierung und Analyse eines hochdichten zufälligen Telekommunikationsnetzwerks (M, April 2017)
- Ein Gibbs-Ansatz für Nachrichtentrajektorien in einem hochdichten Kommunikationsnetzwerk mit mehreren Basisstationen (B, October 2017)
- Informationskapazität in großen zufälligen Kommunikationsnetzwerken (M, December 2017)
- Optimierung des Durchsatzes mit kontinuierlicher Perkolation (B, May 2018)
- Markov Chain Monte Carlo for message routing (M, June 2018).

9. References

Here is the list of publications that arose from the work in the project.

[KT17a] W. König and A. Tóbiás: *A Gibbsian model for message routing in highly dense multi-hop network*, WIAS Preprint No. 2392 (2017), DOI: 10.20347/WIAS.PREPRINT.2392

[KT17b] W. König and A. Tóbiás: *Routeing properties in a Gibbsian model for highly dense multihop networks*, WIAS Preprint No. 2466 (2017), DOI: 10.20347/WIAS.PREPRINT.2466

[CHJ18] E. Cali, C. Hirsch, and B. Jahnel: *Continuum percolation for Cox point processes*, WIAS Preprint No. 2479 (2018), DOI: 10.20347/WIAS.PREPRINT.2479

[HJP18] C. Hirsch, B. Jahnel, and R.I.A. Patterson: *Space-time large deviations in capacity-constrained relay networks*, Lat. Am. J. Probab. Math. Stat. **15** (2018), DOI: 10.30757/ALEA.v15-24

[HJPP17a] C. Hirsch, B. Jahnel, R.I.A. Patterson, and P. Keeler: *Large deviations in relay-augmented wireless networks*, Queueing Syst. (2017), DOI: 10.1007/s11134-017-9555-9

[HJPP16] C. Hirsch, B. Jahnel, R.I.A. Patterson, and P. Keeler: *Large-deviation principles for connectable receivers in wireless networks*, Adv. in Appl. Probab. **48** (2016), DOI: 10.1017/apr.2016.65

[HJPP17b] C. Hirsch, B. Jahnel, R.I.A. Patterson, and P. Keeler: *Traffic flow densities in large transport networks*, Adv. Appl. Probab. **49** (2017), DOI: 10.1017/apr.2017.35

[HJ17] C. Hirsch and B. Jahnel: *Large deviations for the capacity in dynamic spatial relay networks*, WIAS Preprint No. 2463 (2017), DOI: 10.20347/WIAS.PREPRINT.2463

[C+18] E. Cali, T. En-Najjari, N.N. Gafur, C. Hirsch, B. Jahnel, and R.I.A. Patterson: *Percolation for D2D networks on street systems*, IEEE WiOpt (2018), DOI: 10.20347/WIAS.PREPRINT.2479

[K+18] P. Keeler, B. Jahnel, O. Maye, D. Aschenbach, M. Brzozowski: *Disruptive events in high-density cellular networks*, IEEE WiOpt (2018), DOI: 10.20347/WIAS.PREPRINT.2469

10. Data Security

Most of the work was theoretical and did not require any data. The only data relevant to our work were about the coordinates of the Hausvogteiplatz, which is public domain and not very extensive.

11. Press and Public Relations

Does not apply